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Multiple Regressive Model Adaptive Control

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1. Introduction

It is common practice to use linear plant models and linear controllers in the control systems design. Such approach has simple explanation applying to plants with insignificant non-linearity or to those, functioning closely to a working point. But linear controllers, indeed with some modification, are used even for plants with significant non-linearities. Because of several reasons the non-linear controllers have not broad application. First, the linear control theory is well developed; while the non-linear control methods are clear for few engineers in practice. Second, there are some technological and economical difficulties to get high quality study of the process to be controlled in order to build detailed (more precise) non-linear plant model. Third, new ideas in the field of the control theory are continuously realized, which expand the span of the linear control systems applications as an alternative to utilizing complicated models at the expense of troubles of theoretical and practical nature.

During the last years a strategy “separate and rule over” is employed more and more by the researchers when trying to solve complex systems tasks using the principle: “Each complex task can be split into a limited number of simple subtasks in order to solve them independently, thereby formulate the solution of the initial complex task by their particular solutions”. Thus an old idea in the classical works [Wenk & Bar-Shalom, 1980; Maybeck & Hentz, 1987] was revived, so a complex (non-linear and/or time-variant) processes with high degree of uncertainty is represented by a family or a bank of (linear and/or time invariant) models with low degree of uncertainty [Li & Bar-Shalom, 1992; Morse, 1996; Narendra & Balakrishnan, 1997; Murray-Smith & Johansen, 1997].

In fact, the multiple-model adaptive control (MMAC) theory is based mainly on the state space representation via Kalman filters as a tool for static and dynamic estimation of the system model states [Blom & Bar-Shalom, 1988; Li & Bar-Shalom, 1996; Li & He, 1999]. The alternative of implementing multiple-model control using a set of input-output models was the next natural step, even to answer the question: “Why publications in the state space dominate and input-output models are not used for traditional linear control of complex plants, neglecting the fact that the standard system identification delivers basically such type of models”. The researchers in the field of switching control theory are among the

supporters of input-output models in the MMAC [Anderson et al., 2001; Hespanha & Morse, 2002; Hespanha et al., 2003].

A MMAC of time-variant plants using a bank of controllers designed on the base of linear sampled-time models is presented in the next sections. Our research on this topic is on dead-beat controllers (DBC), because on one side the design of DBC is relatively simple and on other side it is appropriate to demonstrate the theoretical development of the multiple-model control based on selecting the DBC order independently on the plant model order [Garipov & Kalaykov, 1991] and on selecting sampling period for the DBC independently on the sampling period of the entire control system. In both aspects the advantage of the DBC is the possibility to express and respectively determine the extreme magnitudes of the control signal through the DBC coefficients. Two approaches to implement the closed loop system are discussed, namely by switching and by weighting the control signal to the plant. A novel solution for MMAC is formulated, which guarantees the control signal magnitude to stay always within given constraints, introduced for example by the control valve, for all operating regimes of the system. Two types of multiple-model controllers are proposed: the first operates at fixed sampling period and contains a set of controllers of different orders, and the second contains a set of controllers of the same fixed order but computed for different sampling periods. Examples of the MMAC are demonstrated and results are compared with the behavior of some standard control schemes.

2. Main principles and concepts in the MMAC

2.1. Modeling the uncertainty in control systems

The most methods for controller design require a good knowledge of controlled plant dynamics or the exact plant model. If this information is incomplete the controller design is under the conditions of *a priori uncertainty* regarding the structure and parameters of the plant model and/or disturbances on the plant. On the other hand, the study of the most industrial processes during their operation is impeded due to equipment aging or failures, operating regimes variations or/and noisy factors changes. And if the *a priori uncertainty* could be justified before the control design, the *a posteriori uncertainty* accompanies the entire control system work. It is obvious that the continuously variation in operating conditions make the controllers function incorrect during the time even in case of exactly known process models.

The historical overview shows various ways of representing the uncertainty in control systems. Limiting the framework to the difference equation as a typical input-output plant description, one can find out that the deterministic time invariant model is substituted in the seventies of 20 century with the stochastic one and the plant dynamics uncertainty is presented by an unmeasured random process on its output, i.e. *the uncertainty is presented as a noise in the output measurements*. When the theory moved the emphasis to time-variant systems in the eighties, this was a sign of recognition that if plant dynamics is changing in time, it can be tracked by estimating the changing model parameters, i.e. *the uncertainty is presented as a noise over the physical model parameters*. Meanwhile there were attempts deterministic interval models to be applied, so the uncertain plant dynamics is presented by a multi-variant model, i.e. *the uncertainty is described as a combination of disturbances to the physical model parameters*. Time-variant and interval models describe with various degrees of

complexity changing plant dynamics. The first type of models can be substituted with the bank of elementary time-invariant models called local models. The second type of models includes a set of time-invariant models for the plant dynamics, every one of which defined within a given range of plant parameters variations. In this case the local models correspond to particular operating regimes or plant states. Nevertheless, for both types of models the following idea is used: a bank of more simple models is used instead of its complicated presentation by a global model. It means that the plant control design of a complex controller can be replaced by a bank of local controllers tuned for every elementary model.

2.2. Multiple-model adaptive control (MMAC)

The core idea to get over the control system uncertainty is to realize a strategy for control of arbitrary in complexity plant by a bank of linear discrete controllers, which parameters depend on the corresponding linear discrete models, presented the plant dynamics at various operating regimes. This strategy is known as *multiple model adaptive control (MMAC)*. The following characteristics are typical for this type of control:

- First, the continuous-time space of the plant dynamics is approximated at limited number of operating regimes. This approach is something other than the indirect adaptive control (well known as self tuning control (STC), where the estimation procedure takes place at each sampling instant, which means that the plant dynamics is examined at practically infinite number of operating points. Hence the MMAC is defined as a new control methodology, which provides new features of the control system by simply using the elements and techniques from the classical control theory and practice.
- Second, MMAC escapes the necessity of on-line plant model estimation. It is true that the bank of local models corresponds to the current plant dynamics at each operating point but these structures are evaluated before the control system starts operating. Hence, MMAC can avoid also all problems of the closed-loop identification compared to the standard indirect adaptive control.
- Third, in case when one exact plant model is not suitable for all operating regimes, the following MMAC approaches can be applied:
 - (a). Multiple-Model Switching Control (MMSC) - Used if the operating regimes are predefined or are quite different. The principle of relay-race control can be observed – each controller of the bank takes independent action in the control system tuned according the best corresponding plant model at the corresponding regime.
 - (b). Multiple-Model Weighting Control (MMWC) - Used if the operating regimes are not known in advance. The plant description is made as combination of the models for other operation regimes or as mixture of limited number of hypothetical models taken from the model bank. The global control is formed by contributions of all local controllers of the bank depending on various weights.

Hence, MMAC is defined as adaptive control, because it uses different combinations of models to describe complex system behavior, thus, even when the plant and controller are time-variant, the controller is designed as being for a time-invariant system.

- Forth, to identify the current operating regime is a specific task to recognize single or a set of performance indices of the control system. A test or number of tests is applied in order to determine some desired conditions (model and plant fit, control system errors,

system performance with respect to a reference model, constraints on signals in the system, etc.), then predefined or prepared in advance solutions for the multiple-model controller behavior is selected. From that viewpoint, the MMAC can be seen as supervisory control as well.

3. Design of MMAC based on a bank of input-output models

3.1. Stages of the design

The multiple model control using bank of controllers tuned under corresponding bank of plant models is a classical control scheme. Usually the model's and controller's sampling periods are the same as the control system sampling period. A block-diagram of the MMACS is given in Fig. 1 for time-variant plant control.

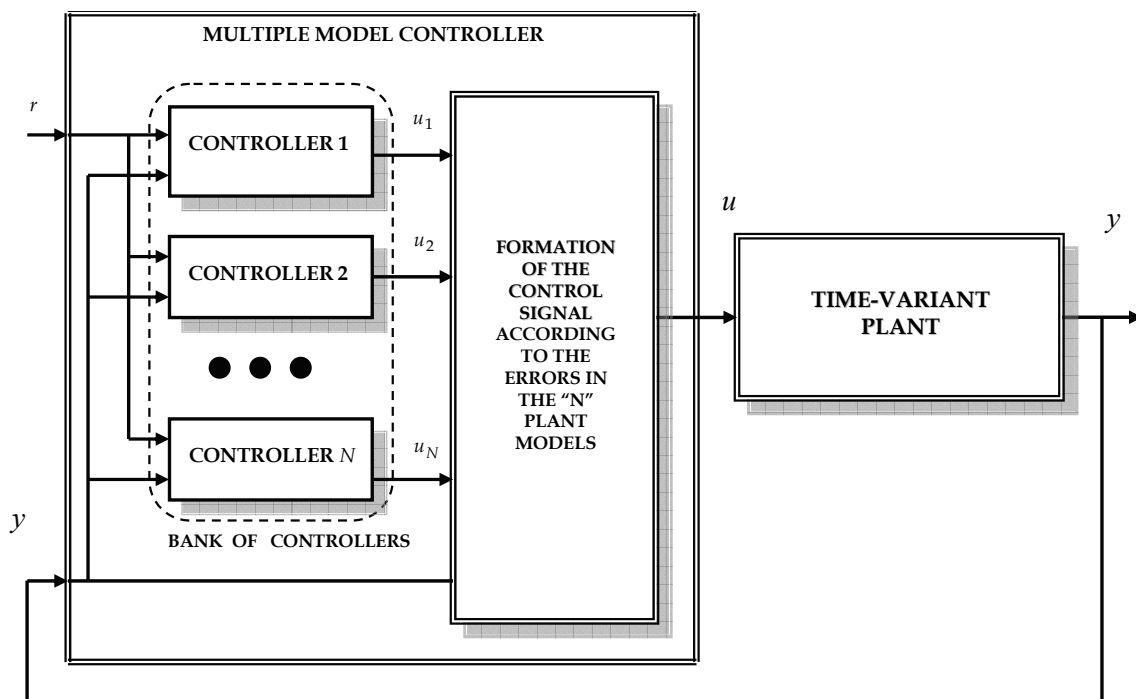


Fig. 1. Structure scheme of the MMACS

The design of MMACS is performed in several stages:

Stage 1. Preliminary choice of a limited set of models, including amount and type of models, estimation of model parameters. Naturally, the MMAC designer aims at a good model, and therefore at a good controller covering a wide range of the system operating conditions. MMACS will act optimally if the model adequately presents the identified plant. When the system is not well studied and it is difficult to obtain a non-linear plant model, MMAC offers the use of a combination of linear models or the choice of the best one among the model set. Such solution is sub-optimal, but acceptable for the prescribed performance criteria. Continuous-time or sampled-time models may be used but the last one is common. The amount of selected models is usually related to the operating condition at which the control system is expected to work.

Stage 2. Preliminary design of sampled multiple-model controller including amount and type of the local controllers and tuning of the controllers' coefficients. The system functional behavior depends mainly on the designed controllers according to the predefined system performance criterion, which is related to the controlled variable $y(\cdot)$. It is accepted that each local controller is tuned according to the corresponding model in the set from Stage 1.

Stage 3. Implementation of the control system, including selection and implementation of the techniques for calculation of the weighting coefficients, and choice of system initial conditions. The weighting technique determines the weights μ_i for the output of each local controller. Usually they depend on values of preferred error signals, for example identification errors, control errors, deviations from a trajectory, etc., which are included into a corresponding criterion such as integral square error ISE, integral absolute error IAE, etc., under given observed time interval. One possible universal type of supervisor to form the control u in a control system is shown in the block diagram on Fig. 1 and is disclosed in details on Fig. 2.

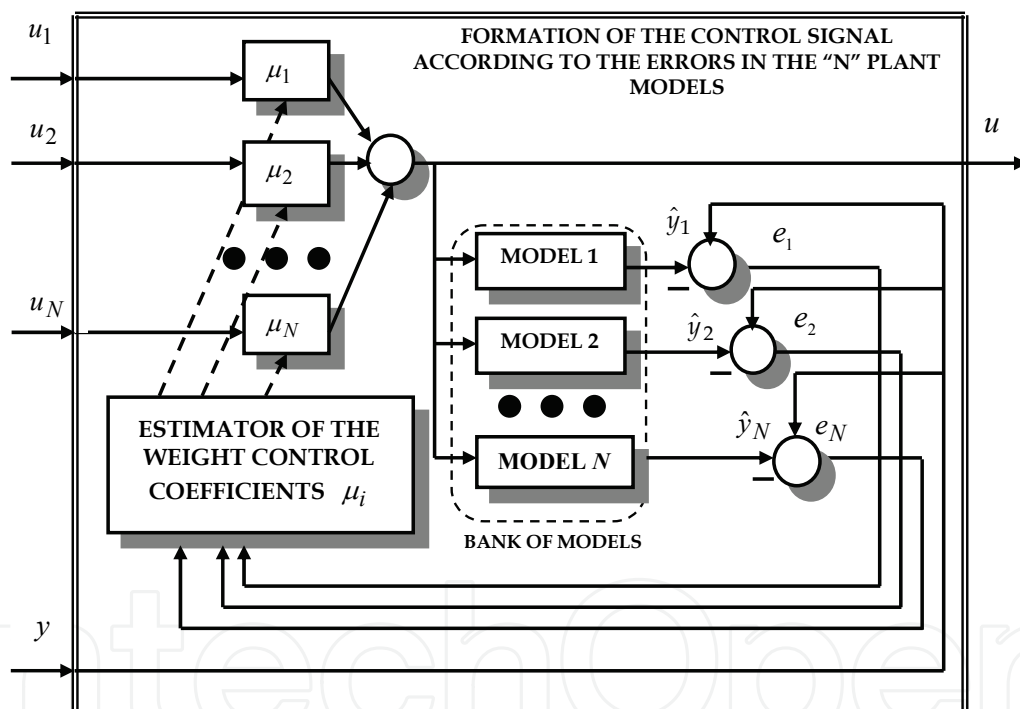


Fig. 2. Detailed block diagram of the MMACS

3.2. Algorithm for output feedback system controller

The general tasks described above can be ordered in the following basic algorithm for discrete MMAC of a continuous-time plant.

Step 1. The number N of the operating plant regimes is specified. The sampling period T_0 for the system is selected, meaning that the signals are measured at time instances kT_0 , $k=0,1,\dots,M$, during the time interval $T=MT_0$. The sampling period is further excluded

from the values index to get shorter notation. The reference signal is defined $r(kT_0) \equiv r(k)$, $k = 0, 1, \dots, M$, $r(0) = 0$. Initial values of the weighting coefficients $\mu_j(0) = 1/N$ are determined. The control system is examined taking the zero initial conditions for the plant and the controller, i.e. $y(0) = 0$ and $u(0) = 0$. Then in the classical system the output signal will be formed answering the following rules:

- ◆ Rule 1. $y(1) = \dots = y(d) = 0$ in case of d sampling periods time-delay in the system ($d \geq 1$ for sampled continuous-time plant using zero-order hold),
- ◆ Rule 2. $y(d+1) \neq 0$, when $r(1) \neq 0$ and $u(1) \neq 0$, if the controller doesn't put its own time-delay into the control system.
- ◆ Rule 3. During the first d sampling intervals the system operates practically without a feedback, therefore certain well known problems in the control might appear.

Step 2. A bank of N discrete-time models is identified using the input-output plant measurements collected at the specified N operating regimes. The suggested models are

$$A_j(q^{-1})y(k) = B_j(q^{-1})u(k - d_j) + e_j(k), \quad (j = 1, 2, \dots, N), \quad (1)$$

where $A_j(q^{-1}) = 1 + a_{j1}q^{-1} + \dots + a_{jn_a}q^{-jn_a}$ and $B_j(q^{-1}) = b_{j1}q^{-1} + \dots + b_{jn_b}q^{-jn_b}$ are polynomials of the unit delay q^{-1} with order, respectively, jn_a and jn_b (more often $jn_a = jn_b = jn$), and d_j presents the delay of the j -th model expresses as integer number of sampling periods. The random signal e_j represents the model estimation error (the mismatch between the physical plant dynamics and its model, the measurement noise or any other disturbances to the plant). When the error is a white Gaussian process, the model parameter estimates are unbiased according the standard least squares method. In case of colored noise the modified least squares methods have to be used in order to get unbiased estimates.

Step 3. A multiple-model controller is designed containing a set of N sampled-time controllers with 2 degrees of freedom and description

$$P_j(q^{-1})u_j(k) = -S_j(q^{-1})y(k) + T_j(q^{-1})r(k), \quad (j = 1, 2, \dots, N), \quad (2)$$

where polynomials $P_j(q^{-1}) = 1 + p_{j1}q^{-1} + \dots + p_{jn_p}q^{-jn_p}$, $S_j(q^{-1}) = s_{j0} + s_{j1}q^{-1} + \dots + s_{jn_s}q^{-jn_s}$ and $T_j(q^{-1}) = 1 + t_{j1}q^{-1} + \dots + t_{jn_t}q^{-jn_t}$ have sizes and parameters according to the selected design method, as well as the corresponding plant model from the bank of the N -th sampled-time models (2). A special case of (3) presents sampled-time controllers with one degree of freedom $S_j(q^{-1}) = T_j(q^{-1}) = Q_j(q^{-1}) = q_{j0} + q_{j1}q^{-1} + \dots + q_{jn_q}q^{-jn_q}$, $r(k) - y(k) = e(k)$, so that

$$P_j(q^{-1})u_j(k) = Q_j(q^{-1})e(k), \quad (j = 1, 2, \dots, N). \quad (3)$$

A cycle for MMACS elements operating at $k = 1, 2, \dots, M$

Start

Step 1. *Functioning of the set of controllers.*

A current control signal is formed at the output of the j -th local controller described by equation (3)

$$\begin{aligned} u_j(k) = & -p_{j1}u_j(k-1) - \dots - p_{jn_p}u_j(k-jn_p) - s_{j0}y(k) - s_{j1}y(k-1) - \dots - s_{jn_s}y(k-jn_s) \\ & + r(k) + t_{j1}r(k-1) + \dots + t_{jn_r}r(k-jn_r) \end{aligned} \quad (4)$$

or by equation (4)

$$u_j(k) = -p_{j1}u_j(k-1) - \dots - r_{jn_p}u_j(k-jn_p) + q_{j0}e(k) + q_{j1}e(k-1) + \dots + q_{jn_q}e(k-jn_q) \quad (5)$$

Step 2. *Weighting control signals of all local controllers.*

A global control signal is calculated by

$$u(k) = \sum_{j=1}^N \mu_j(k-1) u_j(k), \quad (j = 1, 2, \dots, N) \quad (6)$$

The initial values of the weighting coefficients in MMACS can be selected as $\mu(0) = 1/N$, i.e. the weighting mechanism starts with an equal weight of each controller.

Step 3. *Plant response measured.*

The plant output $y(k) = f\{y(k-1), \dots, u(k-1), \dots\}$ is measured.

Step 4. *Supervisory function to form the weighting coefficients at the next cycle.*

Step 4.1. The output $\hat{y}_j(k)$ of each local model is calculated

$$\hat{y}_j(k) = -a_{j1}\hat{y}_j(k-1) - \dots - a_{jn_a}\hat{y}_j(k-jn_a) + b_{j1}u_j(k-d_j) + \dots + b_{jn_b}u_j(k-d_j-jn_b), \quad (7)$$

Step 4.2. The a posteriori residual error $\hat{e}_j(k)$ at each local model output is estimated

$$\hat{e}_j(k) = \{y(k) - \hat{y}_j(k)\} / r(k) \quad (8)$$

Step 4.3. A performance index of each local model is fixed

$$J_j(k) = \hat{e}_j^2(k) \quad (9)$$

Step 4.4. An exponential smoothing is applied to decrease the influence of random factors to the MMACS

$$\bar{J}_j(k) = \lambda \bar{J}_j(k-1) + (1-\lambda)J_j(k), \quad \bar{J}_j(0) = J_j(0), \quad (10)$$

where $\lambda = e^{-4/L}$, L is the number of old values between which the smoothing is done.

Step 4.5. The weighting coefficients for the next cycle of the procedure are calculated

$$\mu_j(k) = \bar{J}_j^{-1}(k) \left[\sum_{j=1}^N \bar{J}_j^{-1}(k) \right]^{-1}, \quad \sum_{j=1}^N \mu_j(k) = 1. \quad (11)$$

End

Alternative step 4.5. If the weighting control expression (12) is exchanged by switching one, then at each sampled-time instant a local controller with index c operates only, which is equivalent to setting the weight of the c -th local controller to be 1, i.e. $\mu_c(k) = 1$. The corresponding plant model is chosen among the set of models according to the threshold value $\bar{J}_c(k)$ in the inequality [Boling et al, 2003]:

$$\bar{J}_c(k) > (1+h) \min_j \{ \bar{J}_j(k) \}. \quad (12)$$

3.3. Test design example of MMACS

Let the continuous time-variant plant be defined as:

$$W_o(p) = \frac{K^{(t)}}{(T_1^{(t)}p+1)(T_2p+1)(T_3p+1)}$$

with time-invariant constants $T_2 = 7.5 \text{ s}$ and $T_3 = 5 \text{ s}$. It is proposed that the observation interval is $T = kT_0$, $k = 0, 1, \dots, M = 300$, $T_0 = 1 \text{ s}$, and the gain $K^{(t)}$ and the time constant $T_1^{(t)}$ evolve as shown on Fig. 3a and Fig. 3b.

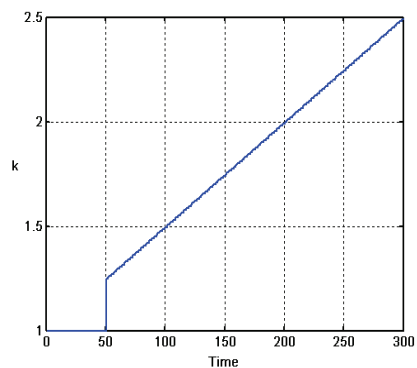


Fig. 3a. Evolution of $K^{(t)}$

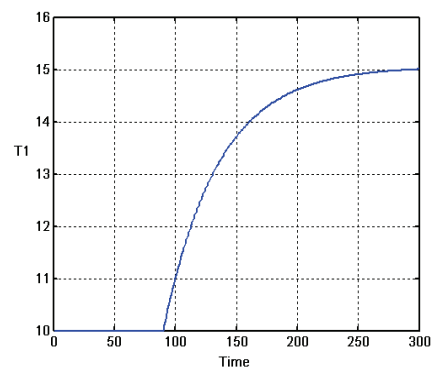


Fig. 3b. Evolution of $T_1^{(t)}$

The MMACS for this plant is tested at 5 operating regimes ($M = 5$). The corresponding primary continuous time-invariant plant models are defined to cover the areas of the parameters' evolution. Five local sampled-time models are calculated to form the bank of models in order to design the local controllers. Then a multiple-model controller is constructed, consisting of five dead-beat controllers each of them tuned for the corresponding model according to the technique described in the next sections. MMACS starts with equal weights $\mu_i(0) \equiv \mu(0) = 1/5$.

Some tests of the designed MMACS are on Fig. 4a (system output and reference), and on Fig. 5a (the behavior of the weighting coefficients for each local controller output). Table 1 demonstrates that MMACS outperforms the other systems. The importance of this is additionally highlighted by the fact MMACS does not use the time-consuming plant identification procedure in real time as a part of self tuning controller.

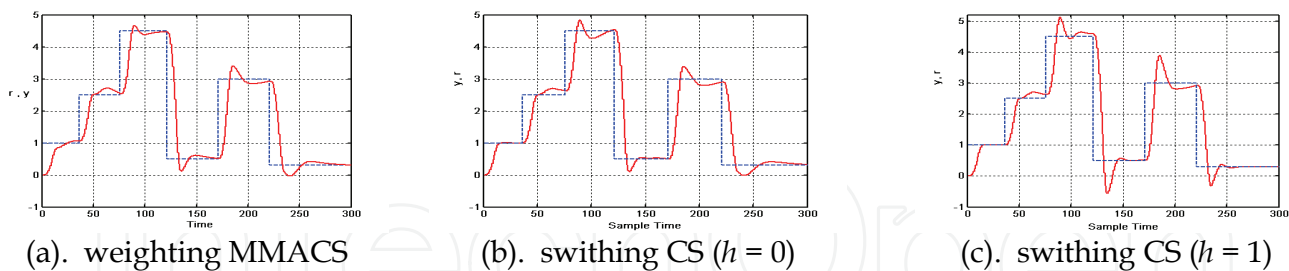


Fig. 4. Output y and reference r

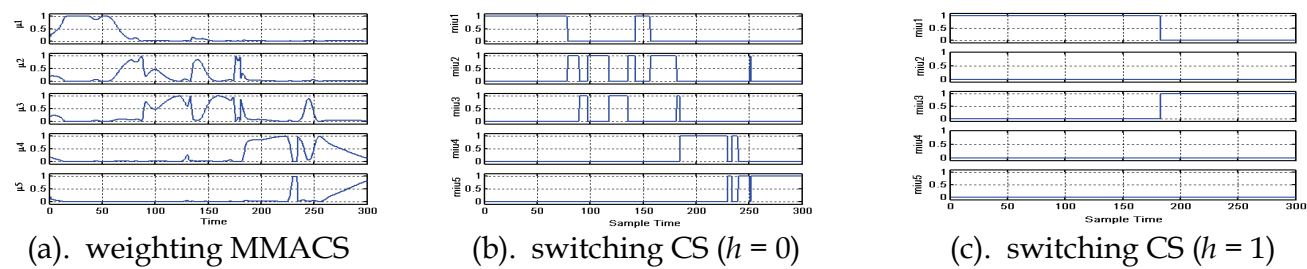


Fig. 5. Weighting coefficients

Type of the control system	A quality measure
MMACS with weighted control u	0.8122
MMACS with switching control u ($h = 0$)	0.8137
MMACS with switching control u ($h = 1$)	0.8161
Classical CS	0.9236
Adaptive CS (STC)	0.8892

Table 1. Mean-square error for comparison

4. Multiple-model adaptive control with control signal constraints

4.1. Introduction

Control systems in practice operate under constraints on the control signal, normally introduced by the control valve. When such constraints are not included in the design of the controller, the system performance differs significantly to the theoretically expected behavior. In case control signal formed by the standard controller is beyond these constraints, it will not be propagated with the required (expected) magnitude according to the unconstrained case. The solution of this problem is offered by the DB controller with increased order, as it adheres to the important principle *“an increased order controller provides decreased magnitude of the control signal”* [Isermann, 1981]. Accordingly, the choice of the order increment that can reduce the influence of the constraints on the control signal is a recommended requirement for the control system designer [Garipov & Kalaykov, 1991].

In control systems with existing constraints on the magnitude of the control signal, linear control at any operating point is feasible in the following two cases:

- (a) When the controller is of sufficiently high order, such that it provides control magnitude not beyond the constraints. Such over dimensioned controller, however, is normally inert and sluggish.
- (b) When the controller is of varying order, which adjusts its coefficients according the necessity to keep the control signal magnitude limited [Garipov & Kalaykov, 1991], but preserving the linear nature of the controller at any operating point of the system.

Equivalent of the system of case (b) is the multiple-model control system with multiplexing DB controllers of various orders is shown on Fig. 6.

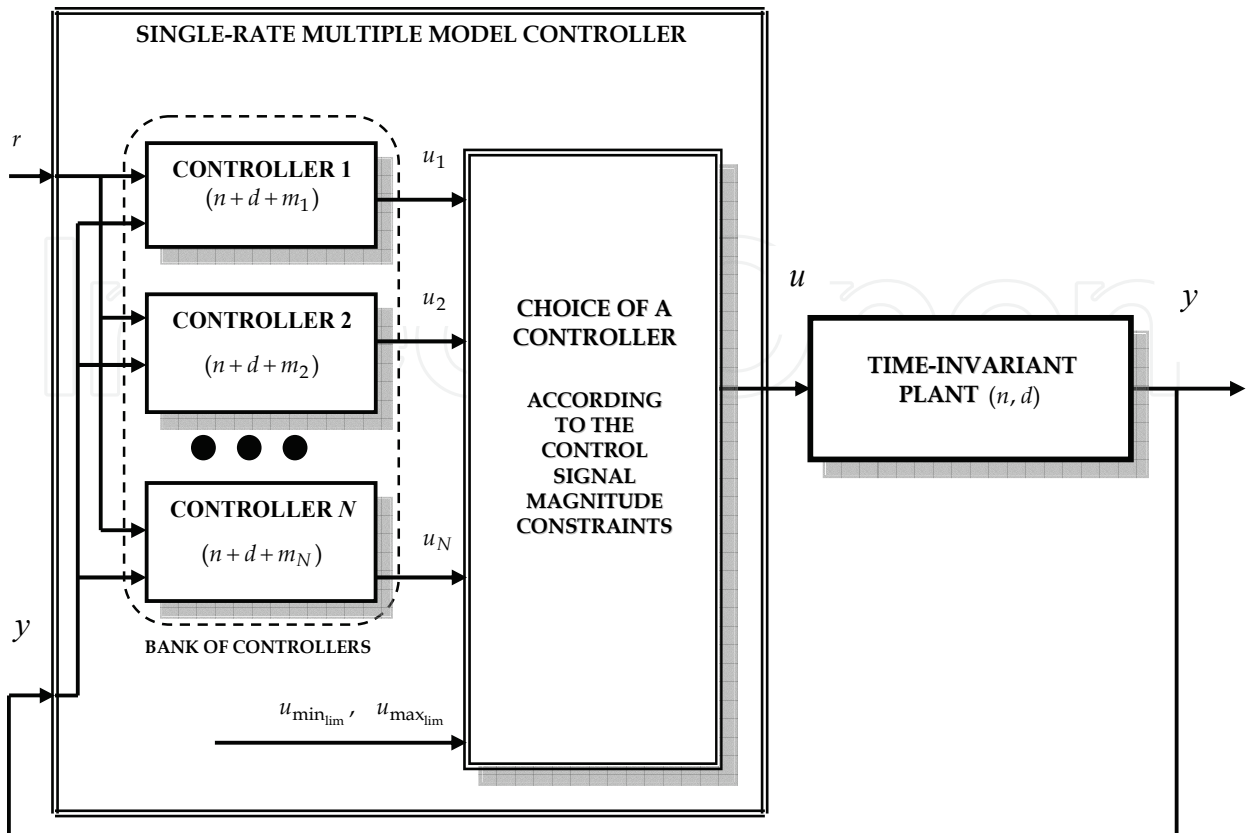


Fig. 6. Single-rate multiple-model system with control signal constraints

The difference to the system described in the previous Section 3, given on Fig. 1, is the selection mechanism, which is based on the requirement the DBC to guarantee a linear control signal within the predefined constraints at any operating point, depending on the desired (possibly stepwise changing only) reference signal.

4.2. Design of DBC of increased order [Garipov & Kalaykov, 1990]

Fundamental property of the DBC is the finite step response time $(n + d)T_0$ of the closed-loop system, where n is the order and d is the time delay of the sampled-data model of the controlled plant. Keeping the sampling period T_0 we can only obtain longer step response by increasing the DBC order. This, however, has the positive effect of decreasing the extreme magnitudes of the control signal, because the energy of the control signal spreads over larger number of sampling intervals (longer time). Thus, the application of DB control can be revived, as presented in the text below.

The design of DBC of increased order is based on the following assumptions:

Assumption 1: The DBC of increased order denoted by DBC $(n + m, d)$ is described by fraction of two polynomials of n_p -th order, where $n_p = n + d + m$ and m means the order increment. When $m = 0$, the DBC is of normal order, when $m = 1$ DBC is of increased by one order [Isermann, 1981], etc.

Assumption 2: At initial conditions the controlled variable $y(0) = 0$, a step change of the reference signal r is applied ($r(0) = r(1) = \dots = 1$) and the step response settles in finite time, i.e.

$$\begin{aligned} y(0) = y(1) = \dots = y(d) = 0, \quad y(d+1) \neq 0, \quad y(k) = r(k) = 1 \text{ when } k \geq n+d+m, \\ u(0) \neq 0, \quad u(k) = u(n+m) \text{ when } k > n+m. \end{aligned} \quad (13)$$

Following the signal behavior after (14), the z-domain images of the respective quantities are

$$Y(z) = \sum_{i=1+d}^{n+d+m-1} y(i)z^{-i} + 1 * \left[\sum_{s=n+d+m}^{\infty} z^{-s} \right], \quad U(z) = \sum_{i=0}^{n+m-1} u(i)z^{-i} + \text{const} * \left[\sum_{s=n+m}^{\infty} z^{-s} \right], \quad R(z) = \frac{1}{1-z^{-1}}.$$

By constructing the following two fractions

$$\frac{Y(z)}{R(z)} = z^{-d}P^{(m)}(z) = \sum_{i=1}^{n+m} p_i z^{-i} = (1-z^{-1})Y(z), \quad \frac{U(z)}{R(z)} = z^{-d}Q^{(m)}(z) = \sum_{i=0}^{n+m} q_i z^{-i} = (1-z^{-1})U(z),$$

the DBC polynomials $P^{(m)}$ and $Q^{(m)}$ are formed. It is not difficult to establish the following **properties of the coefficients of the DBC($n+m, d$)**:

$$\begin{aligned} \sum_{i=1}^k p_i^{(m)} = y(k), \quad k = 1+d, 2+d, \dots, n+d+m \quad \text{and} \quad \sum_{i=1}^{n+m} p_i^{(m)} = y(n+d+m) = 1 \\ \sum_{i=0}^k q_i^{(m)} = u(k), \quad k = 0, 1, 2, \dots, n+m \quad \text{and} \quad \sum_{i=0}^{n+m} q_i^{(m)} = u(n+m) = \text{const}. \end{aligned} \quad (14)$$

The closed-loop control system has a transfer function

$$W_{CL}(z) = \frac{Y(z)}{U(z)} = z^{-d}P^{(m)}(z) = \frac{\sum_{i=1+d}^{n+d+m} p_i z^{n+d+m-i}}{z^{n+d+m}} \quad (15)$$

with a characteristic equation $z^{n+d+m} = 0$, which means that the system has an infinite degree of stability. The properties of $p_i^{(m)}$ coefficients in (15) means the DBC($n+m, d$) guarantees the system steady error to be asymptotically closed to zero (17), because

$$W_{CL}(z)|_{z=1} \equiv z^{-d}P^{(m)}(z)|_{z=1} = \sum_{i=1}^{n+m} p_i = 1. \quad (16)$$

It is not difficult to prove the inherent existence of the same property of the DBC($n+m,d$). Rewriting the DBC transfer function

$$W_C(z) = \frac{1}{W(z)} \frac{W_{CL}(z)}{1 - W_{CL}(z)} = \frac{1}{W(z)} \frac{z^{-d} P^{(m)}(z)}{1 - z^{-d} P^{(m)}(z)} \quad (17)$$

and including the controlled plant transfer function

$$\frac{z^{-d} B(z)}{A(z)} = W(z) = \frac{Y(z)}{U(z)} = \frac{\frac{Y(z)}{R(z)}}{\frac{U(z)}{R(z)}} = \frac{z^{-d} P^{(m)}(z)}{Q^{(m)}(z)}, \quad (18)$$

yield

$$W_C(z) = \frac{Q^{(m)}(z)}{1 - z^{-d} P^{(m)}(z)} = \frac{Q_C(z)}{P_C(z)}. \quad (19)$$

The denominator polynomial in (20) is

$$P_C(z) = 1 - z^{-d} (p_1^{(m)} z^{-1} + p_2^{(m)} z^{-2} + \dots + p_n^{(m)} z^{-n} + \dots + p_{n+m}^{(m)} z^{-n-m}),$$

which can be modified such, that the inherent integral part of DBC($n+m,d$) can be demonstrated

$$P_C(z) = (1 - z^{-1}) \bar{P}(z),$$

where

$$\begin{aligned} \bar{P}(z) = & [(1 + z^{-1} + \dots + z^{-d}) p_1^{(m)} + (1 + z^{-1} + \dots + z^{-d} + z^{-(d+1)}) p_2^{(m)} + \dots + \\ & (1 + z^{-1} + \dots + z^{-(d+n+m-1)}) p_{n+m}^{(m)}] \end{aligned}$$

Hence, the final description of DBC($n+m,d$) becomes

$$W_C(z) = \frac{Q^{(m)}(z)}{(1 - z^{-1}) \bar{P}(z)} = \frac{Q_C(z)}{P_C(z)}, \quad \deg \bar{P}(z) = n + m + d - 1. \quad (20)$$

A matrix based approach for design of DBC of increased order is proposed, which might appear to be relatively complex, but this is compensated by the versatility when constraints on the magnitudes of the control signal are predefined. In this approach we first reformulate (19) in another form of equation that connects the parameters of the plant model and the DBC coefficients, namely

$$A(z)P^{(m)}(z) = B(z)Q^{(m)}(z). \quad (21)$$

Equalizing the respective terms in (22) the following matrix equation can be written

$$X^* \theta = Y^*, \quad (22)$$

where:

$$X^* = \begin{bmatrix} E & \vdots & D_e \\ \dots & \dots & \dots & \vdots & \dots & \dots \\ A_1 & \vdots & -B_1 \\ \dots & \dots & \dots & \vdots & \dots & \dots \\ D_a & \vdots & A_2 & \vdots & D_b & \vdots & -B_2 \end{bmatrix}, \quad Y^* = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \theta = \begin{bmatrix} p \\ q \end{bmatrix}$$

with dimensions of the basic matrices

$$\dim X^* = (2n + m + 1) \times (2n + 2m + 1),$$

$$\dim Y^* = 2n + m + 1,$$

$$\dim \theta = (2n + 2m + 1) \times 1.$$

The block matrices are:

$$A_1 = \begin{bmatrix} a_0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ a_1 & a_0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_n & a_{n-1} & \cdot & a_0 & 0 & \cdot & 0 \\ 0 & a_n & \cdot & \cdot & a_0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & a_n & a_{n-1} & \cdot & a_0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} b_1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ b_2 & b_1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_n & b_{n-1} & \cdot & b_1 & \cdot & \cdot & 0 & 0 \\ 0 & b_n & \cdot & \cdot & b_1 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & b_n & \cdot & \cdot & b_1 & 0 \end{bmatrix}, \quad p = \begin{bmatrix} p_1^{(m)} \\ p_2^{(m)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ p_{n+m}^{(m)} \end{bmatrix}, \quad q = \begin{bmatrix} q_0^{(m)} \\ q_1^{(m)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ q_{n+m}^{(m)} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} a_n & a_{n-1} & \cdot & \cdot & a_1 \\ 0 & a_n & a_{n-1} & \cdot & a_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & a_n \end{bmatrix}, \quad B_2 = \begin{bmatrix} b_n & b_{n-1} & \cdot & \cdot & b_1 \\ 0 & b_n & \cdot & \cdot & b_2 \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & b_n \end{bmatrix}, \quad E = [1 \ 1 \ 1 \dots 1],$$

$\dim A_1 = (n+m) \times (n+m)$, $\dim B_1 = (n+m) \times (n+m+1)$, $\dim A_2 = n \times n$, $\dim B_2 = n \times n$, $\dim E = 1 \times (n+m)$, $\dim p = (n+m) \times 1$, $\dim q = (n+m+1) \times 1$. The dimensions of the null matrices are: $\dim D_e = 1 \times (n+m+1)$, $\dim D_a = n \times m$, $\dim D_b = n \times (m+1)$.

Obviously, the equation (23) represents incomplete system of $(2n+m+1)$ linear equations with $(2n+2m+1)$ unknown parameters. Therefore, to enable the solution m additional equations have to be added, for example m conditions for the elements in the θ vector.

From (15) it is already known that the control signal magnitude at separate time instants depends on the coefficients in the $Q^{(m)}$ polynomial. This can be used to formulate a new concept for design of $DB(n+m,d)$ controller: the unique solution of (23) to be persuaded by appropriate fit of the coefficients q_i with the constraints on the control signal. We complement the already demonstrated by Isermann [Isermann, 1981] principle “*an increased order controller provides decreased magnitude of the control signal*” by a new concept, namely “*flexible tuning of the coefficients q_0, q_1, \dots, q_m can provide linear control at any operating point of the control system*”.

Accordingly, in the proposed method for $DB(n+m,d)$ controller design we introduce a matrix Z , $\dim Z = m \times (n+m+1)$, which augments the incomplete rank equation (23) to the full rank equation

$$X\theta = Y, \quad (23)$$

where the matrices

$$X = \begin{bmatrix} X^* & & \\ \dots & \dots & \dots \\ D_z & \vdots & Z \end{bmatrix}, \quad Y = \begin{bmatrix} Y^* \\ \dots \\ D_y \end{bmatrix}$$

have dimensions $\dim X = (2n+2m+1) \times (2n+2m+1)$, $\dim Y = 2n+2m+1$ and D_z , D_y are zero-blocks with dimensions $\dim D_z = m \times (n+m)$, $\dim D_y = m \times 1$.

The following rules for putting together the elements of the Z matrix are established:

Rule 1. The number of elements in a row corresponds to the number of coefficients in the $Q^{(m)}$ polynomial, as $\deg Q^{(m)} = n+m+1$.

- Rule 2.** The elements can take only a binary value: “0” means that the corresponding coefficient of the $Q^{(m)}$ polynomial exists (i.e. it is nonzero), while “1” means the corresponding coefficient of $Q^{(m)}$ does not exist (i.e. we consider this coefficient as set to zero).
- Rule 3.** Only one value “1” is permitted in a row and it cannot be at the first or last position, because this means change of the degree of $Q^{(m)}$.
- Rule 4.** The nonexistence of the j -th coefficient ($j = 2, 3, \dots, n+m$) in $Q^{(m)}$ (i.e. it is set to zero) means holding of the $(j-1)$ -th value of the control signal, which enables the designer to shape the behavior of the control system.

The proposed methodology is demonstrated below for a $DB(n+m,d)$ controller with $m = 2$ for a third order plant with a delay ($n_a = n_b = n = 3$ and $d = 1$):

$$W(z) = \frac{0.06525z^{-1} + 0.04793z^{-2} - 0.00750z^{-3}}{1 - 1.49863z^{-1} + 0.70409z^{-2} - 0.09978z^{-3}} z^{-1}$$

The controller parameters estimates, allocated in the unknown parameters vector,

$$\theta = \left[p_1^{(2)} \ p_2^{(2)} \ p_3^{(2)} \ p_4^{(2)} \ p_5^{(2)} \ q_0^{(2)} \ q_1^{(2)} \ q_2^{(2)} \ q_3^{(2)} \ q_4^{(2)} \ q_5^{(2)} \right]^T, \\ \dim \theta = (2 * 3 + 2 * 2 + 1) \times 1 = 11 \times 1,$$

can be obtained by solving equation (24), in which

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_0 & 0 & 0 & 0 & 0 & -b_1 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_0 & 0 & 0 & 0 & -b_2 & -b_1 & 0 & 0 & 0 & 0 \\ a_2 & a_1 & a_0 & 0 & 0 & -b_3 & -b_2 & -b_1 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 & 0 & 0 & -b_3 & -b_2 & -b_1 & 0 & 0 \\ 0 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & -b_3 & -b_2 & -b_1 & 0 \\ 0 & 0 & a_3 & a_2 & a_1 & 0 & 0 & 0 & -b_3 & -b_2 & -b_1 \\ 0 & 0 & 0 & a_3 & a_2 & 0 & 0 & 0 & 0 & -b_3 & -b_2 \\ 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 & -b_3 \\ 0 & 0 & 0 & 0 & 0 & z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} \\ 0 & 0 & 0 & 0 & 0 & z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} \end{bmatrix}, \quad Y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dim X = 11 \times 11, \quad \dim Y = (2 * 3 + 2 * 2 + 1) \times 1 = 11 \times 1.$$

Six alternatives for selecting the Z matrix elements, ($\dim Z = 2 \times 6$), are possible as shown below. For some of them the expected behavior of the control signal u is illustrated. The $Q^{(m)}$ coefficients are given in Table 2 and $P^{(m)}$ coefficients are in Table 3.

Alternative 1. $Z = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$, therefore the DB control signal takes values $u(2) = u(1) = u(0)$, as shown on Fig. 7.



Fig. 7. The output and control signal for Alternative 1

Alternative 2. $Z = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, therefore the DB control signal takes values $u(1) = u(0)$ and $u(3) = u(2)$, as shown on Fig. 8.

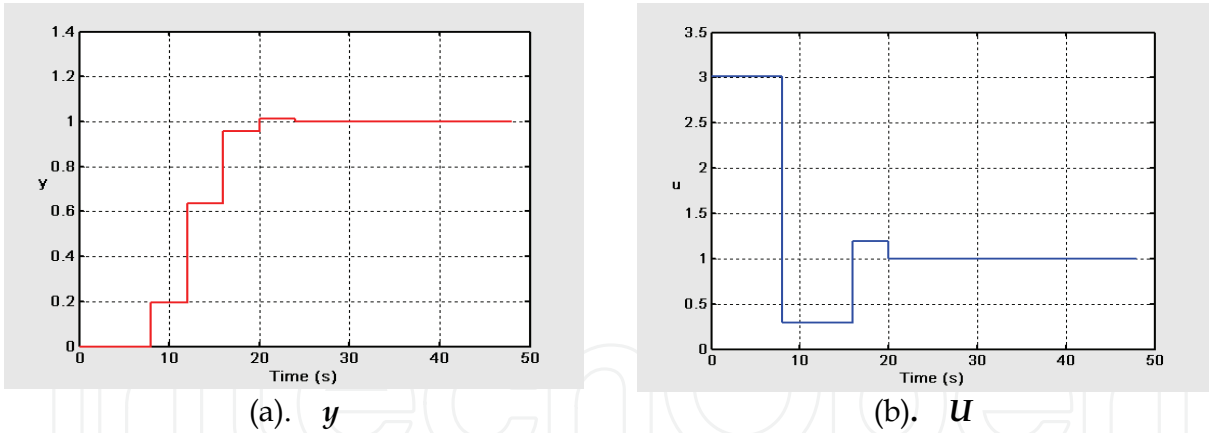


Fig. 8. The output and control signal for Alternative 2

Alternative 3. $Z = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, therefore $u(1) = u(0)$ and $u(4) = u(3)$.

Alternative 4. $Z = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, therefore $u(3) = u(2) = u(1)$.

Alternative 5. $Z = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, therefore $u(2) = u(1)$ and $u(4) = u(3)$.

Alternative 6. $Z = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, therefore $u(4) = u(3) = u(2)$.

Not all alternatives have the same importance. Those, which hold the initial two values $u(0)$ (particularly!) and/or $u(1)$, because these values contribute significantly to the reduction of the control signal magnitude.

Alternative	$q_0^{(2)}$	$q_1^{(2)}$	$q_2^{(2)}$	$q_3^{(2)}$	$q_4^{(2)}$	$q_5^{(2)}$
1	2.34196	0	0	-3.17382	2.19215	-0.36029
2	3.01725	0	2.72848	0	0.90316	-0.19193
3	3.49041	0	-4.64023	2.22379	0	-0.07397
4	5.13095	-4.50996	0	0	0.49397	-0.11496
5	5.94221	-5.82182	0	0.92321	0	-0.04360
6	7.68261	-9.95441	3.29384	0	0	-0.02204

Table 2. Coefficients in the numerator polynomial of the controller

Alternative	$p_1^{(2)}$	$p_2^{(2)}$	$p_3^{(2)}$	$p_4^{(2)}$	$p_5^{(2)}$
1	0.15281	0.34126	0.38626	0.14674	-0.02708
2	0.19687	0.43966	0.31961	0.05828	-0.01442
3	0.22775	0.50861	0.27290	-0.00370	-0.00556
4	0.33479	0.45338	0.18909	0.03138	-0.00864
5	0.38773	0.48600	0.13173	-0.00218	-0.00328
6	0.50129	0.46995	0.03152	-0.00110	-0.00166

Table 3. Coefficients in the denominator polynomial of the controller

The maximal and minimal values of the control signal during the transient response for the given example are collected in Table 4. The *max...min* span can be used as reference values in a criterion to select the appropriate alternative of the DB controller.

m	0	1	2	2	2	2	2	2
Alternative	1	1	1	2	3	4	5	6
$u_{\max} \equiv u(0)$	9.46	3.78	2.34	3.01	3.49	5.13	5.94	7.68
u_{\min}	-4.72	-2.05	-0.83	0.29	-0.15	0.62	0.12	-2.27

Table 4. Extreme values of the control signal for the considered alternatives

4.3. Block “Selection of controller under control signal constraints”

Each local controller submits its computed control signal to this block. Which of them will be transferred as a global control signal to the plant is selected by checking the conditions of getting control signal within the predefined constraints. For time-invariant plants significant changes in the control signal may be obtained due to rapid change of the reference signal or suddenly appearing “overloading” disturbances. Both these factors can be interpreted as step signals, which appear not so often, such that the controller succeeds to stabilize the controlled variable before the appearance of a new disturbance. This assumption aids the explanation about the nature of the logical decisions about the control signal and its constraints.

Let the step change appears at sampled time k causing a system error $|e(k)| = |r(k) - y(k)| \gg \varepsilon$, where ε is a threshold value determining the sensitivity of the algorithm. The local DB controllers with transfer functions

$$W_c(i; z) = \frac{Q(i; z)}{1 - P(i; z)}, \quad Q(i; z) = \{q_0 + q_1 z^{-1} + \dots + q_{nq} z^{-nq}\}_i, \quad P(i; z) = \{p_1 z^{-1} + \dots + p_{np} z^{-np}\}_i,$$

will yield control signals with extreme magnitudes $u_{i \max}, u_{i \min}, i = 1, 2, \dots, N$, which appear at time $k=0$ and $k=m+1$ according the well known property of DBC that $\sum_{j=0}^k q_j = u(k)$

unit step change of the reference

Therefore, the maximum control signal magnitude at the first sampling instant after the step change ($e(k) > \varepsilon$) is equal to the coefficient q_{i0} , ($\{u(0)\}_i = q_{i0} \equiv u_{i0}$). Having in mind this property, the $u_{i \max}$ value of the local control signal right after the step change from:

$u_{i \max} = u(k-1) + (u_{i0} \times e(k)), \quad i = 1, 2, \dots, N$. In parallel the $u_{i \min}$ value at the $(m+1)$ -th

sampling instant after the step change of reference from $\left\{ \sum_{j=0}^{m+1} q_j = u(m+1) \right\}_i \equiv u_{i m+1}$ and

consequently $u_{i \min} = u(k-1) + (u_{i m+1} \times e(k)), \quad i = 1, 2, \dots, N$.

The local DB controller, which complies the constraints, is selected:

- (a) When $e(k) > \varepsilon$ the local controller j among all controllers in the bank is decided according the produced by it maximal value of the control signal, which is less or equal to $u_{\max \lim}$: $u_{j \max} \equiv \max\{u_{1 \max}(k), u_{2 \max}(k), \dots, u_{N \max}(k)\} \leq u_{\max \lim}$. If the additional condition $u_{j \min} \geq u_{\min \lim}$ is satisfied, the selection of controller is confirmed, otherwise first the condition $u_{j \min} \equiv \min\{u_{1 \min}(k), u_{2 \min}(k), \dots, u_{N \min}(k)\} \geq u_{\min \lim}$ is checked and selection confirmed if $u_{j \max} \leq u_{\max \lim}$ is also true.
- (b) When $e(k) < -\varepsilon$ the local controller j is decided by checking first $u_{j \max} \equiv \min\{\text{sign}[e(k)]u_{1 \max}(k), u_{2 \max}(k), \dots, u_{N \max}(k)\} \geq u_{\min \lim}$. If the additional

condition $u_{j\min} \geq \text{sign}(\Delta r)u_{\max\lim}$ is satisfied, the selection of controller is confirmed, otherwise first the condition $u_{j\min} \equiv \max\{\text{sign}(\Delta r)[u_{1\min}(k), u_{2\min}(k), \dots, u_{N\min}(k)]\} \leq u_{\max\lim}$ is checked and selection confirmed if $u_{j\max} \leq \text{sign}(\Delta r)u_{\min\lim}$.

4.4. Single-rate MMDBC under control signal constraint – a test example

Let us take the same continuous control plant given in Section 3.3 and formulate MMDBC containing multiple DBC tuned for the same sampled plant model, but, contrarily to the previous case, having different increments of the order, i.e. each DBC is DB(3+m,1), m=0, 1, 2, 3, 4, 5. The DBC producing extreme values of the control signal within the predefined constraints is activated currently within the MMDBC. Figure 9 represents the performance of the system, where the reference and system output are compared on Fig. 9a, the control signal all the time being within the constraints [-1, 7.5] on Fig. 9b. Figure 9c shows how the DBC order increment m varied when stepwise changing the reference signal.

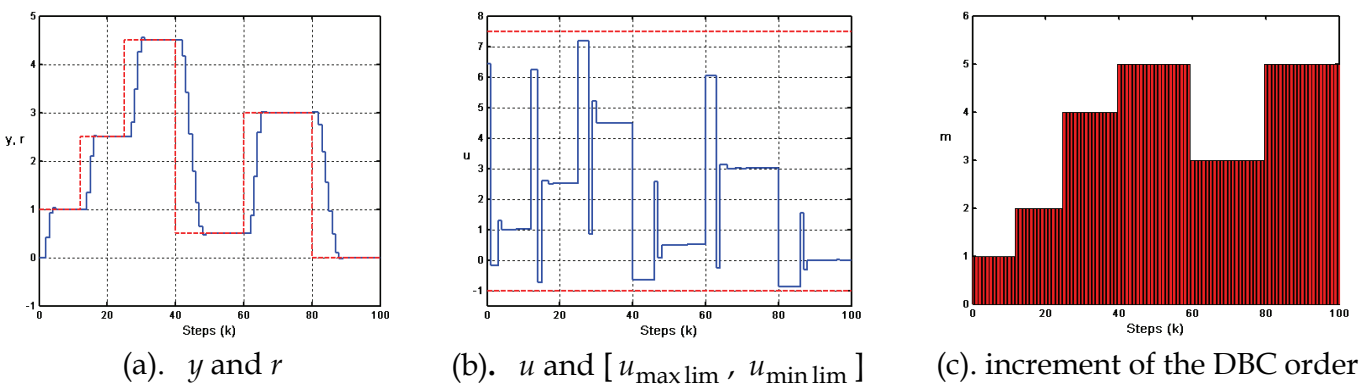


Fig. 9. Single-rate MMDB control system with control signal constraints

For comparison of the proposed MMDBC, a standard DB control system with fixed increments, namely DBC(3+0,1) and DBC(3+1,1), is demonstrated on Fig. 10, where one can see worse performance under the same test conditions. This confirms the advantage of our MMDBC approach.

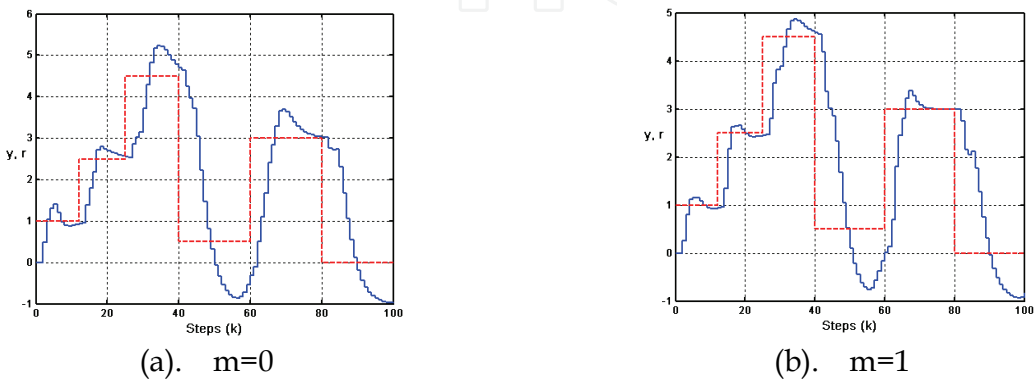


Fig. 10. System behavior y and r in control system with a standard fixed order DBC

4.5. Design of two-rate DB control system

The assumption in the sampled-data control systems theory is to define a sampling period T_0 , which is valid for the entire closed-loop system (let call it T_0^{CL}) and for the controller itself (let call it T_0^C). In other words $T_0^{CL} = T_0^C = T_0$. It is known that $T_0^{CL} = T_0$ should be small enough for achieving nearly continuous-time system behavior, or at least the Shannon sampling theorem should be satisfied. However, it is also known that a small sampling period $T_0^C = T_0$ yields large magnitudes of the control signal, which go beyond the physical constraints of the control valve, i.e. the nonlinear nature of the system becomes dominating. Therefore, certain lower bound of $T_0^C = T_0$ should be considered.

Garipov proposed in [Garipov, 2004] a control scheme for DB control of a continuous plant based on the following postulates:

- First, in order to form a sampled control signal for the continuous plant with a sampling period $T_0^C > \tau$ identical to the sampling period of the entire control system, the system error at the controller input to be sampled with the same sampling period as the controller is sampled. This means the control feedback has to be implemented in an inner closed loop containing the controller and a sampled-time model of the plant, both with the same sampling period. In other words, both blocks have to operate with synchronous sampling rate. A normal performance of the DBC is expected even when the sampling period of the controller is $T_0^P > \tau$.
- Second, in order to close the loop around the physical plant, an outer feedback loop is provided as well, based on the mismatch between the physical plant output (with all types of disturbances, measurement noise, etc.) and a sampled model of the plant (which is noise-free). The mismatch error could be considered much closer to zero when the sampling period of this second model of the plant is very small (nearly continuous system). This implies the recommendation the closed-loop sampling period to be selected always smaller, i.e. $T_0^{CL} < T_0^C$. A normal performance of the DBC in this two-rate control system is expected.

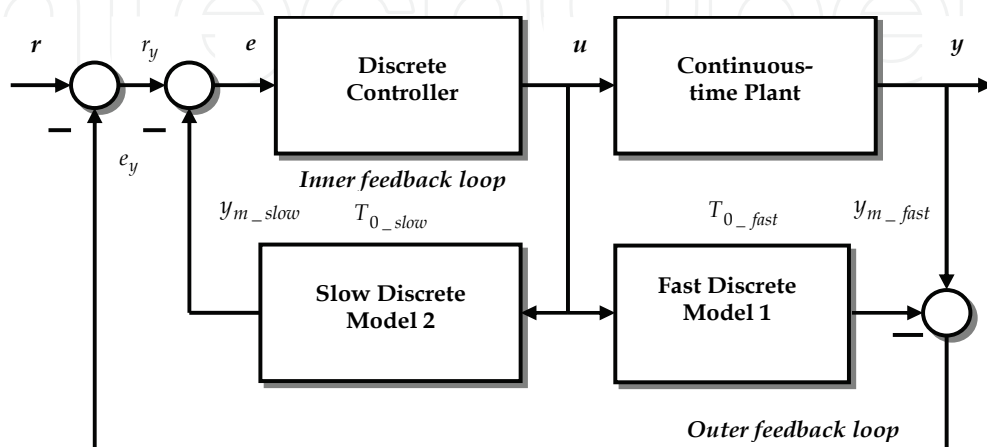


Fig. 11. Two-rate control system

The block diagram of two-rate control system is shown on Fig. 11. Two sampled-time models of the same continuous-time plant are included, namely Fast Discrete Model with sampling period $T_{0_fast} \equiv T_0^{CL}$, which is in parallel to the plant, and Slow Discrete Model with sampling period $T_{0_slow} \equiv T_0^C > T_{0_fast}$, which is used for the design of the controller. The step response of such system is demonstrated on Fig. 12, where one can identify the small sampling period $T_0^{CL} = 1$ sec, while the controller operates at sampling period $T_0^C = 8$ sec.

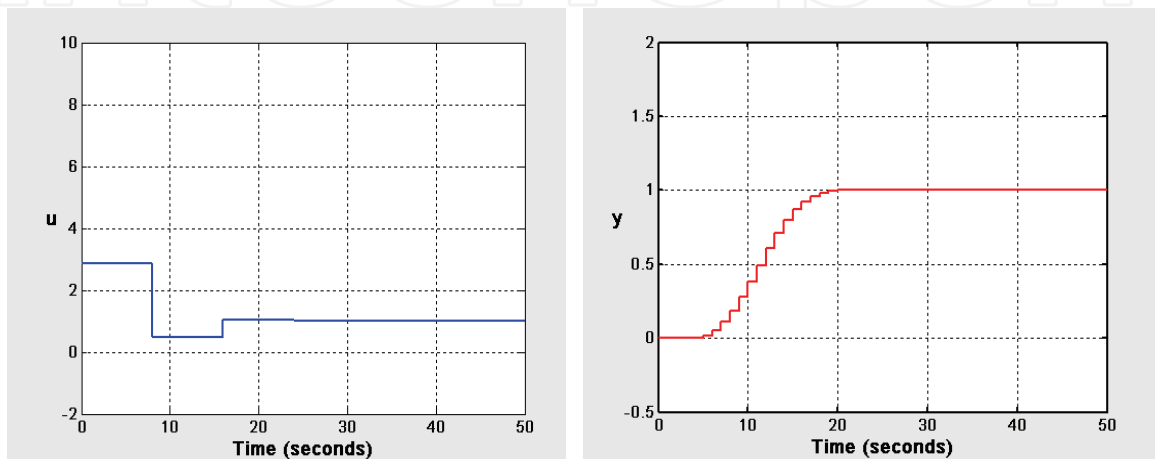


Fig. 12. Modified two-rate system for $T_0^{CL} = 1$ s and $T_0^C = 8 * T_0^{CL} = 8$ s

4.6. Design of a multi-rate DBC

The main idea of a multiple-model adaptive DB controller, in which every local DBC operates at a different sampling period that is not equal of the sampling period of the closed-loop system, is implemented in the block diagram on Fig. 13.

Algorithm

- Step 1. The continuous-time plant model is identified.
- Step 2. The small sampling period T_0 of the control system is selected and the respective sampled-time plant model obtained.
- Step 3. A number of sampled-time models of the plant with different sampling periods $T_0^{(i)}$, $i = 1, 2, \dots, N$, are computed and the corresponding DBC obtained. The respective extreme values of the control signal for each model are calculated.
- Step 4. The control signal constraints $[u_{\min \text{ lim}}, u_{\max \text{ lim}}]$ are defined and desired profile of the reference signal $r(k)$, $k=0, 1, 2, \dots, M$, is specified (for analysis of the system performance).

Step 5. The MMDBC system is started, then the described in Section 4.3 block “Selection of controller” at every step change of the reference signal checks and selects the DBC with the least sampling period providing extreme values of the control signal within the defined constraints to be active.

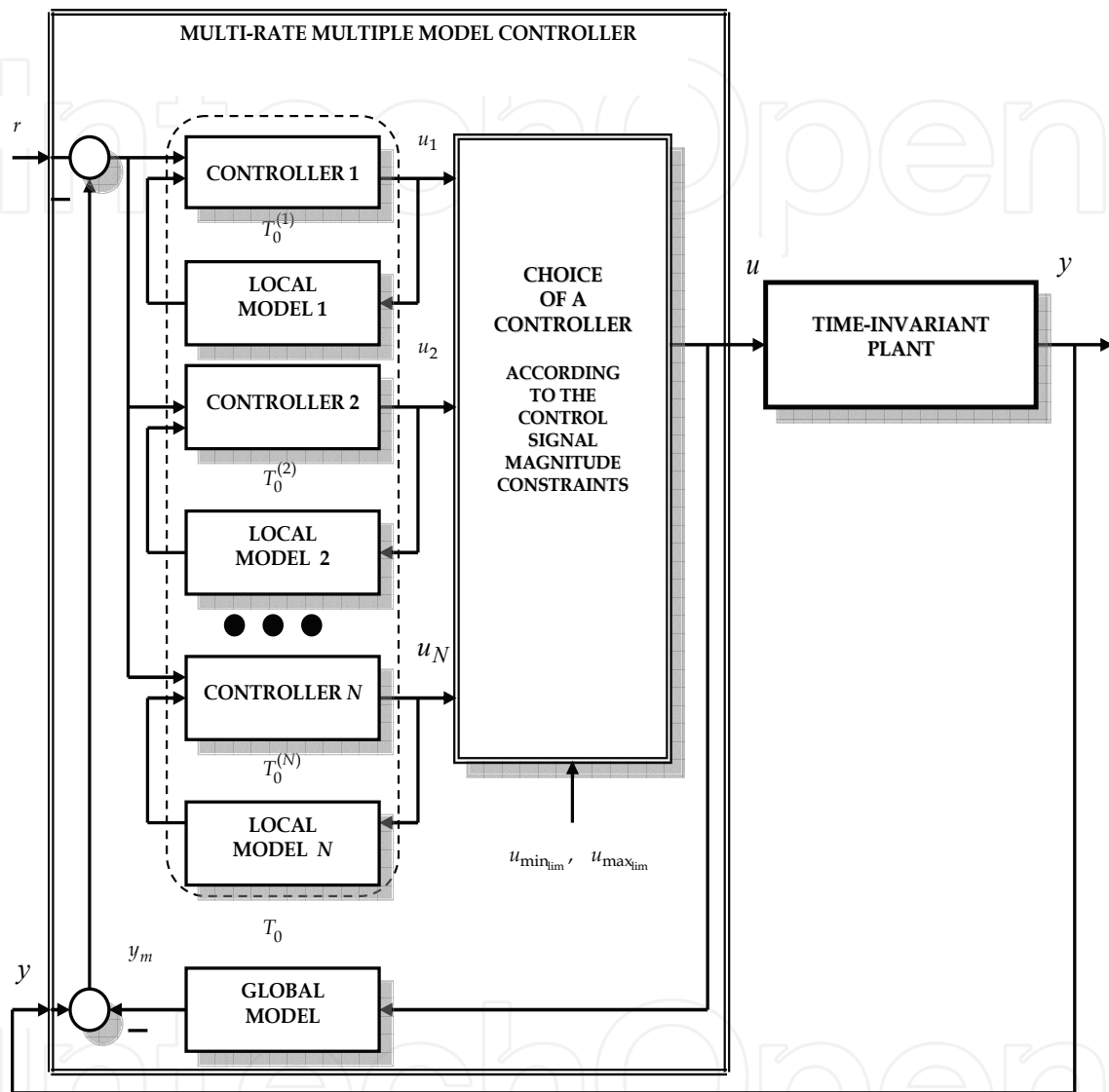


Fig. 13. Multi-rate multiple model control with control signal constraints

4.7. Multi-rate MMDBC under control signal constraint – a test example

Let us take the same continuous control plant given in Section 3.3 and formulate multi-rate MMDBC containing multiple DBCs tuned for the same sampled plant model, but, contrarily to the previous case, obtained at different sampling periods $T_0^{(i)} = 4, 6, 8, 10, 12, 14$ и 18 sec. The Fast Discrete Model (Fig. 11) is obtained for sampling period $T_0 = 0.1$ sec. Figure 14 represents the performance of the system, where the reference and system output are

compared on Fig. 14a, the control signal all the time being within the constraints $[-1, 7.5]$ on Fig. 14b. Figure 14c shows which model and respective DBC was selected, namely the corresponding value of the sampling period T_0 when stepwise changing the reference signal.

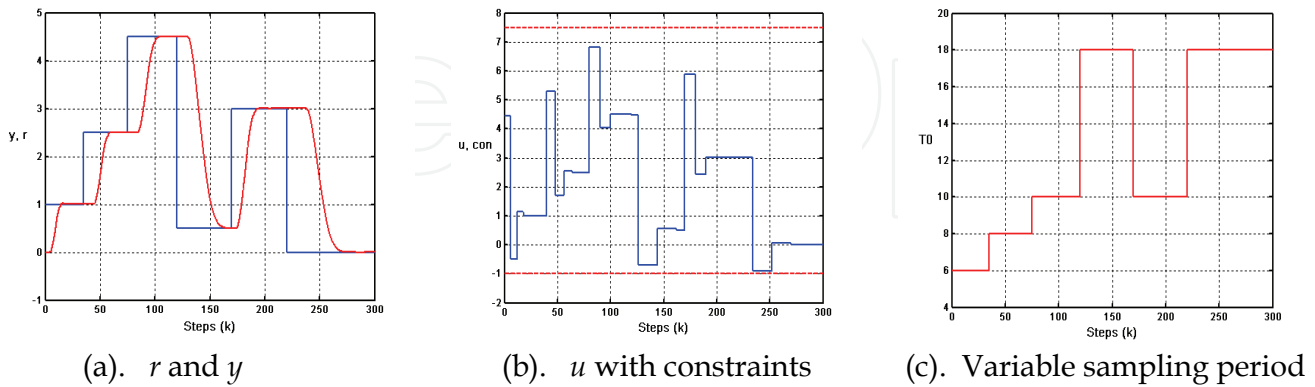


Fig. 14. Multi-rate MMDB control system with control signal constraints

For a comparison, a standard DB control system is demonstrated on Fig. 15 for three different sampling periods $T_0^{(i)} = T_0$, where one can see the poor performance under the same test conditions. This confirms the advantage of our multi-rate MMDBC approach.

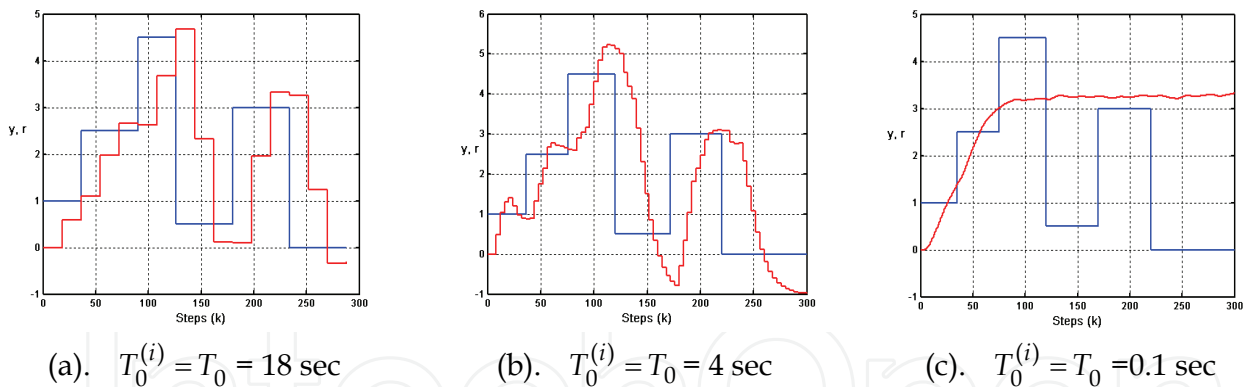


Fig. 15. System behavior y and r in control system with a standard DBC at various T_0

5. Conclusion

The essence of the ideas applied to this text consists in the development of the strategy for control of the arbitrary in complexity continuous plant by means of a set of discrete time-invariant linear controllers. Their number and tuned parameters correspond to the number and parameters of the linear time-invariant regressive models in the model bank, which approximate the complex plant dynamics in different operating points. Described strategy is known as Multiple Regressive Model Adaptive Control (MRMAC), and the implemented control system is known as Multiple Regressive Model Adaptive Control System (MRMACS). Its scheme is very traditional but attention is paid mainly on the novel

algorithm in the supervisory block that forms the final control action if the regimes of the plant are not previously known.

The existence of control signal constraints by the control valve clearly indicates the needs to guarantee a control magnitude that always fits within the control constraints for all operating regime of the system. A novel design procedure is proposed to tune dead-beat controllers (DB) with arbitrarily increased order so the closer is the operating point to the constraints the bigger should be the DB controller order. As the plant operating point continuously changes, the switched MRMAC DB controller of minimal order have to be select in order to satisfy the control signal constraints. The supervisor logical action is shown and tests are made for complex simulation plants. The same task can be solved if the DB order remains fixed but the control signal magnitude is reduced by the switched MRMAC DB controller of arbitrarily discrete time-interval in order to satisfy the control signal constraints. In this case a new multi-rate MRMACS scheme is prepared.

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